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On dielectric membranes

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Abstract: By placing a collection of membranes in 11 dimensional background supergravity six form field and considering the non-linear action of multiple membranes we obtain the potential for transverse scalar fields of membranes at leading order and study some possible vacuum configurations. We find that the system has a stable vacuum in which is a formation of membranes into fuzzy three sphere.

Keywords: D-branes, M-Theory

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Contents

1 Introduction

When a neutral material put in an external electric field the electric charges separate from each other and form an electric dipole. Similar phenomena exists in string theory where there are extended objects such as D_p branes and higher rank antisymmetric form fields as charges of these objects. In particular, When a set of neutral D branes put in an external antisymmetric background field one observes the "polarized" D branes which are expanded into a higher dimensional world volume theory [1].

One may naturally expect such phenomena for membranes in M-theory. There are some efforts to studying this in the context of M-theory [2] where it was used some idea such as considering massive ABJM theory [3] or Basu-Harvey equation [4] as boundary conditions of membranes. But recently, some wonderful works was done in understanding the dynamics of multiple $M2$ branes. Firstly, it was found an action [5] which was based on an strange algebra, Lie-3 algebra, which is expected to describe the low energy dynamics of multiple membranes. Based on this action, the authors of [6] found a non-linear action for $M2$ branes which in low energy limit reduces to that of [5]. Beside, the Lie-3 algebra valued fields allows one to write down an action for the coupling of supergravity background form fields C_3 and C_6 to world volume of membranes [7].

So, by using the nonlinear action of multiple membranes and the action of couplings of background form fields to world volume of membranes one can study the effects of placing multiple membranes in these external fields. The aim of this note is to study it in details.

In particular, we will find that when a collection of membranes are is placed in a constant background C_6 fields the configuration of membranes has a trivial vacua in which the potential $V(X) = 0$ and all fields are represented by some commuting matrices. But, the theory still has a nontrivial vacua in which has lower energy and so is stable in comparison with commuting one. Such vacuum is a configuration of membranes which are formed in fuzzy three sphere S^3 and are ended on five branes.

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2 On triple product

First of all, we briefly introduce triple product.[1] All fields in BLG theory are valued in a Lie-3 algebra and can be expanded in terms of generators of the algebra as[2],[3] $X^I = X^I_a T^a$ in which the generators T^a construct the following product

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d, \tag{2.1}$$

where $f^{abc}{}_d$ are some structure constants and obey a fundamental identity

$$f^{cde}{}_g f^{abg}{}_h = f^{abc}{}_g f^{gde}{}_h + f^{abd}{}_g f^{gec}{}_h + f^{abe}{}_g f^{cdg}{}_h, \tag{2.2}$$

It has been done a lot of efforts to finding solutions of (2.2), see for example [5, 7–11]. An interesting solution was presented in [8–10] in which by decomposing $T^a = \{T^-, T^+, T^a\}$ one may demand

$$\begin{aligned} &[T^-, T^+, T^a] = 0, && [T^a, T^b, T^c] = f^{abc} T^- \\ &[T^+, T^a, T^b] = [T^a, T^b] = f^{ab}{}_c T^c && \end{aligned} \tag{2.3}$$

In this fashion, the T^- mode completely decouples from the theory and one obtains a theory with ordinary Lie algebra generators T^a which is lifted by T^+.

3 Non-linear action for $M2$ branes

Proposed bosonic part of non-linear Lagrangian for multiple membranes for gauge group U(N) is given by

$$\begin{aligned} \mathcal{L}_{M_2} =& -T_2 STr\left((det(S))^{1/4}\sqrt{-det\left(\eta_{\mu\nu} + \frac{1}{T_2}\tilde{D}_\mu X^I \tilde{R}^{IJ} \tilde{D}_\nu X^J\right)}\right) \\ &+\frac{1}{2}STr\left(\epsilon^{\mu\nu\lambda}\left(B_\mu + \frac{\partial_\mu X_+.X - X_+.D_\mu X}{X_+^2}\right)\left(F_{\nu\lambda} + \frac{1}{T_2}\tilde{D}_\mu X^I \tilde{P}^{IJ}\tilde{D}_\nu X^J\right)\right) \\ &+(\partial_\mu X^I_- - Tr(B_\mu X^I))\partial^\mu X^I_+ - Tr\left(\frac{X_+.X}{X_+^2}\hat{D}_\mu X^I \partial^\mu X^I_+ - \frac{1}{2}\left(\frac{X_+.X}{X_+^2}\right)^2 \partial_\mu X^I_+ \partial^\mu X^I_+\right) \end{aligned} \tag{3.1}$$

where A_μ, B_μ, X^I are in adjoint representation of U(N) and X^I_-, X^I_+ are singlet under U(N), and

$$\begin{aligned} M^{IJK} &\equiv X^I_+[X^J, X^K] + X^J_+[X^K, X^I] + X^K_+[X^I, X^J] \\ \hat{D}_\mu X^I &= D_\mu X^I - X^I_+ B_\mu\,, \qquad D_\mu X^I = \partial_\mu X^I + i[A_\mu, X^I], \end{aligned} \tag{3.2}$$

[1] Note that the main results of this paper can be obtained without the assumption of triple algebra on the theory.

[2] In this paper we use the indices as $a, b, \ldots = +, -, 1, 2, \ldots, Dim\ \mathcal{G}$; $\mu, \nu, \ldots = 0, 1, 2$; $\hat{I}, \hat{J}, \ldots = 0, 1, \ldots, 10$ and $I, J, \ldots = 3, 4, \ldots, 10$.

[3] And similarly, one can expand the gauge fields and spinor fields in terms of the generators.

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The above Lagrangian is invariant under global SO(8) transformation and under U(N) gauge transformation associated with the A_μ. There is also another transformation associated with the B_μ gauge field which leaves the Lagrangian invariant

$$\begin{aligned} \delta_B X^I &= X_+^I \Lambda\,, & \delta_B B_\mu &= D_\mu \Lambda\,, \\ \delta_B X_+^I &= 0\,, & \delta_B X_-^I &= Tr(X^I \Lambda) \end{aligned} \tag{3.3}$$

It is important to note that the equation of motion for X_-^I gives $\partial_\mu \partial^\mu X_+^I = 0$. Gauging the shift symmetry $X_-^I \to X_-^I + C^I$ as [12] by introducing a new field C_μ^I and rewriting $\partial_\mu X_-^I$ as $\partial_\mu X_-^I - C_\mu^I$, then the equations of motion of the new fields give $\partial_\mu X_+^I = 0$ which means that $X_+^I = v^I$ for some constant v^I.

4 Membranes coupled to fluxes

The coupling of the antisymmetric fields C_3 and C_6 to world volume of $M2$ branes was given in [7] as

$$\begin{aligned} \mathcal{L}_{MCS} &= \lambda_1 \epsilon^{\lambda\mu\nu} C_{IJK} STr(T^a T^b T^c) D_\lambda X_a^I D_\mu X_b^J D_\nu X_c^K \\ &\quad + \lambda_2 \epsilon^{\lambda\mu\nu} C_{IJKLMN} STr([T^d, T^e, T^f] T^a T^b T^c) X_d^I X_e^J X_f^K D_\lambda X_a^L D_\mu X_b^M D_\nu X_c^N \end{aligned} \tag{4.1}$$

Following [1], and assuming all fields are valued in a *non-associative algebra*, the above Lagrangian may be written in a generalized form as

$$S_{MCS} = \mu_2 \int \left(P \left(e^{i\lambda < \mathbf{i}_X \mathbf{i}_X \mathbf{i}_X >} \Sigma C_{(n)} \right) \right) \tag{4.2}$$

where $\mathbf{i}_X$ denotes the interior product by $X^{\hat{I}}$ as a vector in transverse space and we define the operator $< \mathbf{i}_X \mathbf{i}_X \mathbf{i}_X >$ as

$$< \mathbf{i}_X \mathbf{i}_X \mathbf{i}_X > = < X^{\hat{I}}, X^{\hat{J}}, X^{\hat{K}} >,$$

and the associators is defined as [5]

$$< X^{\hat{I}}, X^{\hat{J}}, X^{\hat{K}} > = (X^{\hat{I}}.X^{\hat{J}}).X^{\hat{K}} - X^{\hat{I}}.(X^{\hat{J}}.X^{\hat{K}}), \tag{4.3}$$

The λ is a constant with dimension $\frac{1}{length^3}$. Defining the triple product as

$$\begin{aligned} [X^{\hat{I}}, X^{\hat{J}}, X^{\hat{K}}] &= < X^{\hat{I}}, X^{\hat{J}}, X^{\hat{K}} > + < X^{\hat{J}}, X^{\hat{K}}, X^{\hat{I}} > + < X^{\hat{K}}, X^{\hat{I}}, X^{\hat{J}} > \\ &\quad - < X^{\hat{I}}, X^{\hat{K}}, X^{\hat{J}} > - < X^{\hat{J}}, X^{\hat{I}}, X^{\hat{K}} > - < X^{\hat{K}}, X^{\hat{J}}, X^{\hat{I}} >, \end{aligned} \tag{4.4}$$

and expanding (4.2) in power of λ one finds

$$\begin{aligned} S_{MCS} &= \mu_2 \int STr \left(P \left(\frac{1}{6} C_{\hat{I}\hat{J}\hat{K}} DX^{\hat{I}} \wedge DX^{\hat{J}} \wedge DX^{\hat{K}} \right) \right) \\ &\quad + i\lambda \mu_2 \int STr \left(P \left(\frac{1}{216} [X^{\hat{I}}, X^{\hat{J}}, X^{\hat{K}}] C_{\hat{I}\hat{J}\hat{K}\hat{L}\hat{M}\hat{N}} DX^{\hat{L}} \wedge DX^{\hat{M}} \wedge DX^{\hat{N}} \right) \right). \end{aligned} \tag{4.5}$$

Obviously for $I, J = \hat{I}, \hat{J} = 3, 4, \ldots, 9$ one can reproduce (4.1). Note that, although we define (4.2) by assuming the non-associativity of fields, one still can write the action (4.5) by demanding that all fields are valued in a triple product algebra only without using the non-associativity.

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First of all, recalling (2.3) and using the gauge symmetry of the BLG action [12], to have the gauge choice $X^I_- = 0$ and supposing that the form fields C_3 and C_6 are not depend on x^I_+ the MCS term of the full Lagrangian doesn't change the solution[4] $X^I_+ = v^I$. So, one may rewrite

$$M^{IJK} = \left[X^I, X^J, X^K\right]. \tag{5.1}$$

From the non-linear Lagrangian the potential for the scalar is given by

$$-T_2 STr\left((det(S))^{1/4}\right) \tag{5.2}$$

which can be expanded as [6]

$$(det(S))^{1/4} = 1 - \frac{1}{12T_2} M^{IJK} M^{IJK} + \ldots \tag{5.3}$$

For the MCS term one has, at lowest order, the following expression for the coupling of C_6 flux to $M2$ branes world volume

$$\begin{aligned}
& i\lambda\mu_2 \int STr\left(\frac{1}{216}\epsilon^{\mu\nu\lambda} M^{IJK} C_{IJK\mu\nu\lambda}(x^\mu, X^I) + \frac{1}{72}\lambda\epsilon^{\mu\nu\lambda} M^{IJK} C_{IJKL\nu\lambda}(x^\mu, X^I) D_\mu X^L\right) \\
&\quad = i\lambda\mu_2 \int d^3x STr \epsilon^{\mu\nu\lambda} M^{IJK}\left(\frac{1}{216}\left(C_{IJK\mu\nu\lambda}(x^\mu) + \lambda X^L \partial_L C_{IJK\mu\nu\lambda}(x^\mu) + \ldots\right)\right) \\
&\quad\quad + i\lambda\mu_2 \int d^3x STr \epsilon^{\mu\nu\lambda} M^{IJK}\left(\frac{1}{72}\left(\lambda C_{IJKL\nu\lambda}(x^\mu) D_\mu X^L + \ldots\right)\right),
\end{aligned} \tag{5.4}$$

where in the second line we use the nonabelian Taylor expansion of C_6 form field. Now, consider that $M2$ branes are placed in a constant nontrivial $F_7 = dC_6$ field strength as

$$F_{\mu\nu\lambda IJKL} = \begin{cases} -\frac{f}{3\lambda^2\mu_2}\epsilon_{\mu\nu\lambda} f_{IJKL} & \text{for } I,J,K,L = \{3,4,5,6\} \\ 0 & \text{otherwise} \end{cases} \tag{5.5}$$

where f is a constant with dimension $(length)^2$. From the above assumptions, the first term in (5.4) equals to zero.[5] By integrating by part of second term and noticing the fact that

$$F_{\mu\nu\lambda IJKL} = \frac{1}{48}\partial_\mu C_{\nu\lambda IJKL} - \frac{1}{36}\partial_I C_{\mu\nu\lambda JKL} \tag{5.6}$$

one obtains for the second and third terms

$$-i\lambda^2\mu_2 \int d^3x Tr\left(\frac{1}{6}\epsilon^{\mu\nu\lambda} M^{IJK} X^L F_{\mu\nu\lambda IJKL}\right) \tag{5.7}$$

So, the potential for the scalars reads as

$$V(X) = -\frac{1}{12} STr\left(M^{IJK} M^{IJK}\right) - \frac{i\lambda^2\mu_2}{6} STr\left(\epsilon^{\mu\nu\lambda} M^{IJK} X^L F_{\mu\nu\lambda IJKL}\right) \tag{5.8}$$

[4] In general, C_6 may have a dependence on X^I_+. If so, then our discussion is true only at leading order.
[5] This is true for constant background field or infinite dimensional algebra.

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Substituting (5.5), recalling $\delta X^I_+ = 0$ and demanding $\delta V(X)/\delta X^I = 0$ yields the equation

$$[M_{IJK}, X^J, X^K_+] - iff^{IJKL}M_{JKL} = 0, \tag{5.9}$$

which is the equation for extrema of $V(X)$.

There is a trivial solution for this equation in which all fields are represented as diagonalized matrices as

$$X^I = \begin{matrix} x^I_1 & 0 & 0 & \ddots \\ 0 & x^I_2 & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ \ddots & 0 & 0 & x^I \end{matrix}. \tag{5.10}$$

This is because of that from the commuting matrices (5.10) we have $M^{IJK} = 0$ which solves the equation (5.9).

As a nontrivial solution we use the following ansatz

$$M_{IJK} = 2iRf_{IJKL}X^L, \tag{5.11}$$

which solves (5.9) if $R = \frac{f}{2}$. So, in this case we have the following equation

$$[X^I, X^J, X^K] = iff^{IJKL}X_L. \tag{5.12}$$

which has a simple solution

$$X^I = \sqrt{f}T^I, \tag{5.13}$$

Since the background field (5.5) breaks the SO(8) symmetry to SO(4) $\times$ SO(4) one may naturally choose the structure constant to be $f_{IJKL} = \epsilon_{IJKL}$ [5].

So, the solution (5.13) represents a fuzzy three-sphere [14] with radius $r^2 = \Sigma(X^I)^2$. By defining $C = \Sigma(T^I)^2$ and noticing that since C is the central of Lie-3 algebra (2.1) one can choose it to be a constant operator, we have

$$r^2 = fC. \tag{5.14}$$

Evaluating the potential $V(X)$ using the solutions (5.13) we have

$$V(X) = -\frac{3}{2}f^3C \tag{5.15}$$

which is lower than that was obtained from the commuting solution (5.10), and means that the commuting solution is unstable and the system goes towards the formation of fuzzy S^3 configuration.

6 Summary

By placing a system of N D_p branes in an external background form field causes that the system would has a vacuum in which is noncommutative an stable against of the commutative one [1]. Similarly, in M-theory, if a collections of membranes would be in an external C_6 form field the system has a vacuum in which membranes are polarized due to field strength effect and are formed into a fuzzy S^3 sphere. This can be interpreted as the formation of spherical branes ended on five-brane.

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